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ABSTRACT

The central objective of factor analysis is to explain the greatest amount of variance in a data set with the smallest number of factors. Higher-order analysis is an invaluable tool that offers the benefit of parsimony provided by first-order analysis with the opportunity to make data-based generalizations beyond the first-order. Higher-order analysis provides a hierarchical framework that better honors the reality with which many phenomena in the social sciences are perceived. Interpretation of higher-order factors requires careful understanding and consideration on the part of the individual researcher. A step-by-step discussion of a real factor analysis is provided to make computer-based results more clear. (Contains 1 figure, 12 tables, and 8 references.) (SLD)

Running head: HIGHER-ORDER ANALYSIS

Higher-Order Factor Analysis: An Introductory Primer

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Abstract

The central objective of factor analysis is to explain the greatest amount of variance in a data set with the smallest number of factors. Higher-order analysis is an invaluable tool that offers the benefit of parsimony provided by first-order analysis with the opportunity to make data-based generalizations beyond the first-order. Higher-order analysis provides a hierarchical framework that better honors the reality with which we perceive many phenomena in the social sciences. Interpretation of higher-order factors requires careful understanding and consideration on the part of the individual researcher.

Factor analysis is a useful technique for managing and interpreting data with many variables. Reducing the number of variables in an analysis to a smaller number of factors facilitates understanding of the data and allows for greater generalization. Higher-order analyses present additional perspectives on data and opportunities for increased generalization. Hetzel (1996) provides an excellent review of the basic concepts in factor analysis.

Higher-order analysis makes sense conceptually when we consider that many phenomena are considered to exist in a hierarchical structure. For instance, the idea of general intelligence (g) can be conceptualized as subsuming both verbal IQ and performance IQ. Verbal IQ and Performance IQ in turn each subsume several Wechsler subtests, and each subtest subsumes several individual items (Gray, 1997). Thus, higher-order analysis seems to represent our perceptions of reality more accurately than first-order analysis alone.

The the present paper provides a conceptual basis for understanding higher-order analysis and elucidates the interpretations that can be made from such analyses. Although a conceptual, rather than a mathematical, framework of factor analysis is presented here, a step-by-step discussion of a real factor analytic example is provided to help make computer-based results more clear.

Review of First-Order Analysis

To review, the objective of factor analysis is to explain the maximum amount of variance in a set of measured or observed variables with the smallest number of synthetic or latent factors, or latent constructs. The relationships among measured variables are expressed in a matrix of associations, such as a correlation matrix or a variance-covariance matrix. Like regression analysis, factor analysis is an example of the general linear model and therefore yields a set of weights that are applied to the measured variables to obtain scores on the latent factors (Vidal, 1997). The weights in factor analysis are called factor pattern coefficients. The factor pattern coefficients are analogous to the weights in regression analysis (Hetzel, 1996; Vidal, 1997).

Factor analysis also yields a factor structure matrix, which is composed of factor structure coefficients, that represent the bivariate correlations between each variable and each one of the factors. The factor structure coefficients are analogous to the structure coefficients in regression analysis (Hetzel, 1996; Vidal, 1997). When the factors are uncorrelated, the factor pattern matrix is the same as the factor structure matrix.

The factor pattern and the factor structure matrices together provide information from which the factors can be identified or interpreted. Typically, factor pattern and factor structure coefficients with magnitudes greater than .60 are considered to be high and coefficients with magnitudes greater than .30 are considered to be moderately high (Hetzel, 1996). Interpretation of factors, however, should be based on convergence of information from the relevant coefficients and information from other relevant sources (Hetzel, 1996).

Each measured variable in a factor analysis has a communality coefficient (h^2) that equals the sum of the squared structure coefficients for that variable. The communality coefficients range from 0 to 1 and represent the amount of variance in each measured variable that is reproduced by the latent factors as a set. Each factor, or latent construct, in a factor analysis has an eigenvalue which represents the variance in the original data matrix that is reproduced by each of the factors. Eigenvalues range from 0 to the number of variables. In Principal Components Analysis (PCA), the sum of the squared structure coefficients for a factor equals the eigenvalue for that factor. The eigenvalue can be converted into an effect size statistic by dividing the eigenvalue for a factor by the number of measured variables. In PCA, the sum of the eigenvalues is equal to the sum of the communality coefficients and can be divided by the number of variables to yield an effect size statistic that represents the portion of variance from the original data matrix that is reproduced by all the factors as a set.

Upon examination of the relevant statistics, the decision of which factors to retain can be made according to several rules. Examples of factor retention rules include eigenvalue greater than one, scree test, tests of statistical significance, Minimum Average Partial (MAP), and parallel analysis. For more information concerning factor retention methods, refer to Hetzel (1996) or Stevens (1996). Knowledgeable researchers should use an approach based on theory and personal values as well as computer results.

Typically, after factors are extracted, the first factor reproduces the greatest amount of the variance. Factor rotation can be helpful by spreading the variance more evenly across the factors and thereby clarifying the factor structure. Factor rotation can be accomplished either orthogonally (yielding uncorrelated factors) or obliquely (yielding correlated factors). Examples of computerized orthogonal rotation procedures include Varimax and Quartimax; examples of computerized oblique rotation procedures include Promax and Oblimin. After rotation, factors are ready for interpretation by the researcher.

Imagine a test of 200 items. Analyzing scores on each, separate item could be a difficult and time-consuming process. With factor analysis, however, a set of factors could be extracted that would allow for a more efficient analysis of the data. For instance, six factors may be extracted that may be called, "vocabulary," "written clarity," "reading comprehension," "quantitative concepts," "arithmetic speed," and "mathematic analysis." Each of the six factors would be expected to explain a portion of the variance in some items. Instead of attempting to make sense of 200 items separately, we can interpret scores on only 6 factors. This is the general purpose of factor analysis.

It may happen that a degree of generalization beyond the six first-order factors is desired. For this purpose, higher-order factors may be extracted.

Higher-Order Analysis

> The first-order analysis is a close-up view that focuses on the details of the valleys and the peaks in mountains. The second-order analysis is like looking at the mountains at a greater distance, and yields a potentially different perspective of the mountains as

> constituents of a range. Both perspectives may be useful in facilitating understanding of data. (Thompson, 1990)

In other words, first-order factors provide a high degree of accuracy, but a low degree of generalization. Second-order factors offer a lower degree of accuracy, but a higher degree of generalization.

Higher-Order Factor Extraction

The higher-order factor analytic process is described as follows. First, the first-order factors must be rotated obliquely, yielding correlated factors in the form of a factor pattern coefficient matrix (P_{vxf}). The resulting correlated factors make up a factor x factor correlation matrix (R_{fxf}) that itself is then used as the matrix of associations, or input, for the second-order factor analysis. From the factor x factor matrix of associations, second-order factors (P_{fxh}) are extracted and a retention rule is applied to determine the number of factors.

An important note about factor retention rules in higher-order analysis must be made. Most methods of factor retention that are used for first-order factors can also be used for second-order factors. The exception to this rule is statistical significance testing. Statistical significance testing is inappropriate for use with higher-order factors because the sampling distribution of correlation coefficients for the first-order factors will vary according to the rotation procedure used (Gorsuch, 1983). Thus, informed researchers should use the eigenvalue $\geq$ 1 rule, scree test, or some other extraction rule at the second-order level.

Returning to our previous example, we may imagine that two possible second-order factor names would be "verbal proficiency" and "quantitative proficiency." We could then rotate the higher-order factor matrix ($\mathbf{P_{fxh}}$), factors by higher-order factors, obliquely to reveal correlated second-order factors. The second-order factor by second-order factor correlation matrix could potentially be used to extract third-order factors. In our previous example, a possible third order factor may be named "Intelligence".

The process of higher-order factor extraction continues until oblique rotation yields uncorrelated factors, or until only one factor is extracted. Typically, factors beyond second or third order are rare. Figure 1 is a graphical representation of first, second, and third order factors for our example.

Interpretation of Higher-Order Factors

One common mistake in factor analysis is to base one's interpretations of higher-order factors on the first-order factors (Thompson, 1990). This practice is essentially "basing interpretations upon interpretations" (Gorsuch, 1983, p. 245). When we extract first-order factors, our aim is to remove the variance that is not important or useful in explaining our object of interest. Variance that is not useful in explaining first-order factors, may be useful in explaining second-order factors. Our object of interest changes from one step to the other, so it would be senseless to limit the amount of variance under consideration to that which was useful in describing the first-order factors. A better approach to the interpretation of second-order factors is to use information given by the variables themselves (Thompson, 1985; 1990). The same holds true for the interpretation of third-order factors and beyond.

Three methods for interpreting higher-order factors using information from the original variables have been developed and will now be discussed. First, Gorsuch (1983) suggested that the first-order factor pattern matrix ($\mathbf{P}_{vxf}$) can be multiplied by the orthogonally rotated higher-order factor pattern matrix ($\mathbf{P}_{fxh}$). This multiplicative process yields a variable-by-higher-order factor matrix of factor pattern coefficients ($\mathbf{P}_{vxh}$).

Second, Thompson (1990) suggested that researchers use Gorsuch's (1983) rule, but apply a Varimax rotation procedure to the resulting matrix (P_{vxh}). Thompson reasoned that, because rotation is used to clarify other factor structures, it seems appropriate to employ orthogonal rotation to clarify interpretations of matrix (P_{vxh}).

Third, the Schmid-Leiman (1957) solution is another method for interpreting higher-order factors. This procedure allows for the simultaneous interpretation of both orders of factors with respect to the observed variables. The Schmid-Leiman solution residualizes (removes) the variance from the first-order factors that is present in the second-order factors, thereby orthogonalizing the first and second-order factors to each other. The following heuristic example should help to make these methods and the process of higher-order analysis more clear.

Example Using "SECONDOR"

This example is based entirely on Thompson's (1990) analysis of dissertation data. Thompson developed the FORTRAN program, SECONDOR, to compute higher-order principal components analyses. The program also offers various methods of factor interpretation. This example is presented, with permission, to illustrate the step-by-step process of higher-order analysis and to facilitate understanding of higher-order results.

The matrix of associations, or input, used in this example is a correlation matrix of 24 variables and is presented in Table 1. The first row of values for each variable represents the correlation between that variable and variables 1 to 12. The second row for each variable represents the correlation between that variable and variables 13 to 24.

A principal components analysis was conducted, and 24 factors were extracted. The eigenvalues are presented in Table 2. According to the eigenvalue $\geq$ 1 rule, only six factors were retained. Table 3 presents the first-order principal components matrix and h^2. Remember that the values presented in this matrix are analogous to the BETA weights in regression analysis. Because the factors have not undergone oblique rotation, and are therefore orthogonal to each other, the principal components matrix represents <u>both</u> the factor pattern matrix (BETA) weights and the factor structure matrix (structure coefficients).

The next step in the analysis is to apply an oblique rotation procedure to the factor pattern matrix in Table 3. The Promax method of oblique rotation was used for this example, and the resulting factor pattern matrix is presented in Table 4. Because an oblique rotation procedure was used, the factors are now correlated, and the factor pattern matrix must be interpreted in conjunction with the factor structure matrix presented in Table 5.

The factor correlation matrix is presented in Table 6. This matrix shows the correlations between each of the first-order factors. If the factors were uncorrelated, there would still be ones on the diagonals of the matrix, but there would be zero, values off the diagonals.

The factor correlation matrix presented in Table 6 was then used as the matrix of associations, or input, for the second-order factor analysis. The second-order eigenvalues are presented in Table 7. Given the eigenvalue $\geq$ 1 rule, two second-order factors were retained. The second-order factor matrix is presented in Table 8. The rows in Table 8 represent the 6 first-order factors and the columns represent the second-order factors. If this was a first-order analysis, the rows would represent the 24 variables and the columns would represent the 6 first-order factors.

The second-order factor matrix was rotated orthogonally to redistribute variance and facilitate interpretation, as reported in Table 9. If there were more factors, and a theoretical basis for doing so, the factor matrix could have been rotated obliquely and third-order factors could possibly have been extracted. Varimax was the orthogonal rotation procedure used in this example.

Now that the higher-order factors have been identified, interpretation becomes the central issue. Remember that it is inappropriate to base interpretations of second-order factors on interpretations of first-order factors. Better practice is to use a rule such as Gorsuch's (1983) method in which the first-order, obliquely-rotated factor pattern matrix (Table 4) is multiplied by the second-order, orthogonally rotated factor matrix (Table 9). The resulting product matrix is presented in Table 10.

Once multiplication of the two matrices is accomplished, the trace for the second-order variables is interpretable with respect to the variables themselves. In other words, if the trace for second-order Factor A (5.25) is divided by the number of variables, we can say that 21.9% of the variance in Factor A is explained by the variance of the variables.

Table 11 presents a Varimax rotation of the product matrix presented in Table 10. Remember that Thompson (1990) suggested that this rotation procedure be applied to the product matrix before interpreting the second-order factors with respect to the variables. Notice that the distribution of trace appears more balanced after the rotation procedure. This difference is due to the rotation, which distributes the variance more equally across the factors.

Table 12 presents the Schmid-Leiman (1957) solution for this example. The variable numbers and names are listed in the first column; the second-order factors head the next two columns; the first-order factors head the next six columns; and the last column contains the h^2 values. The trace is listed at the bottom. Using Table 12, we are able to simultaneously interpret the first and second-order factors in relation to the variables.

Notice that second-order Factor A appears to be explained mostly by variables 1, 4, 8, 12, 18, 19, 21, and 23. Judging by the names of these variables, we might interpret second-order Factor I to represent "intellect." Second-order factor B appears to be explained mostly by variables 7, 10, and 20. Given these variable names, we might interpret second-order Factor II to represent "warmth." Keep in mind that the sign, positive or negative, of the values in the columns is important.

Notice that the trace for the second-order factors is the same as it was for Gorsuch's solution, but the trace for the first-order factors is less than their original eigenvalues. This reduction in trace occurred because the Schmid-Leiman solution orthogonalizes the first-order factors to the second-order factors, so that the shared variance is taken out of the first-order factors. In this example, the second-order factors appear to dominate the factor space, so we know that we are getting a good deal of information from the second-order factors.

Although the second-order factors provide much information, it may still be important to interpret the first-order factors in relation to the variables. First-order Factor III, for example, appears to be explained by variables 2, 6, 7, 9, and 14 and may be interpreted to represent undirectedness. Most of the variables important to first-order Factor III, however, are not important to the second-order factors. Note that in Table 9, the communality coefficient (h^2) associated with Factor III is considerably low. This is consistent with the relative lack of importance of first-order Factor III at the second-order level. Researchers must make judgments about interpreting factors at either the first- or second-order or both.

Summary

This paper has presented a step-by-step illustration of the higher-order factor analytic process. Several interpretation procedures have been reviewed and the benefits of each discussed. Interested students are referred to McClain (1996) for another example using Thompson's (1990) FORTRAN program.

Given the hierarchical nature of many phenomena in the social sciences, higher-order analyses often appear to be preferable to first-order analyses used alone. Researchers must, however, be careful and responsible in making interpretations from their results. It is simply <u>not</u> acceptable to make interpretations of higher-order factors directly from the interpretations of first-order factors. This uninformed approach is completely unnecessary in light of the availability of the several interpretation aids as described previously. It is hoped that this paper has contributed to understanding of higher-order analysis and will encourage researchers to make more thoughtful decisions when interpreting higher-order results.

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Figure 1

First, Second, and Third-Order Factors

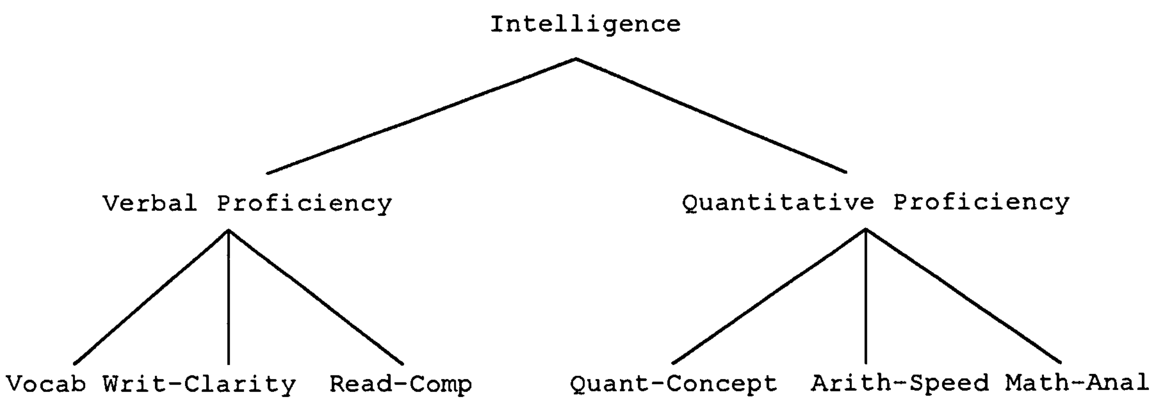

Table 1

Correlation Matrix (Input for Example 1)

1 Intelligent	1.00000	-0.19527	0.35909	0.46019	0.16791	-0.06781	-0.00672	0.37907	-0.17960	0.18324	0.26472	0.34101
	0.17932	-0.18006	0.20436	0.21243	0.14418	0.42960	0.45603	0.08795	0.29710	0.16065	0.30276	0.07474
2 Undirected	-0.19527	1.00000	-0.20057	-0.06301	-0.03932	0.14934	0.19499	-0.17585	0.13339	-0.01455	-0.11005	-0.14293
	0.01078	0.16971	-0.08923	-0.07562	-0.09964	-0.20783	-0.19373	-0.05522	-0.11401	-0.00476	-0.12306	0.05676
3 Honest	0.35909	-0.20057	1.00000	0.45272	0.36218	-0.00329	-0.14299	0.43454	-0.04999	0.38803	0.33674	0.51858
	0.23059	-0.00134	0.38474	0.50338	0.36227	0.41474	0.35462	0.22199	0.26814	0.05753	0.32306	0.22534
4 Scholarly	0.46019	-0.06301	0.45272	1.00000	0.22060	-0.06406	0.07826	0.43621	0.01832	0.10594	0.33152	0.37941
	0.40604	-0.07720	0.13418	0.20220	0.11365	0.55420	0.38408	0.05740	0.35350	0.26112	0.26595	-0.01909
5 Personable	0.16791	-0.03932	0.36218	0.22060	1.00000	0.08289	-0.26066	0.28817	-0.05845	0.40162	0.17708	0.33603
	0.05277	-0.05650	0.29997	0.41362	0.31630	0.15521	0.17123	0.22732	0.20262	0.08867	0.14681	0.34277
6 Easy	-0.06781	0.14934	-0.00329	-0.06406	0.08289	1.00000	0.07763	-0.14798	0.08447	0.12953	-0.08229	-0.06862
	-0.04074	0.35183	0.09027	0.11231	0.00051	-0.09738	-0.01790	0.25119	-0.18404	-0.09930	-0.05840	0.12354
7 Distant	-0.00672	0.19499	-0.14299	0.07826	-0.26066	0.07763	1.00000	-0.12823	0.22806	-0.41278	0.04782	-0.16334
	0.09002	0.24103	-0.25763	-0.27665	-0.25542	-0.05918	-0.14746	-0.27991	0.04706	0.16632	-0.08734	-0.37224
8 Informed	0.37907	-0.17585	0.43454	0.43621	0.28817	-0.14798	-0.12823	1.00000	-0.07443	0.28243	0.22652	0.57457
	0.27448	-0.14837	0.23532	0.34434	0.29557	0.42188	0.44228	0.01958	0.33554	0.20489	0.36140	0.07638
9 Docile	-0.17960	0.13339	-0.04999	0.01832	-0.05845	0.08447	0.22806	-0.07443	1.00000	-0.05276	0.05245	-0.08962
	0.00316	0.28390	-0.01664	-0.02408	-0.06813	-0.00716	-0.07693	-0.06852	-0.00863	0.12362	-0.05419	0.00078
10 Caring	0.18324	-0.01455	0.38803	0.10594	0.40162	0.12953	-0.41278	0.28243	-0.05276	1.00000	0.18484	0.47481
	0.06740	0.02136	0.72002	0.66607	0.49481	0.07539	0.36108	0.27555	0.04285	-0.15746	0.33226	0.54008
11 Systematic	0.26472	-0.11005	0.33674	0.33152	0.17708	-0.08229	0.04782	0.22652	0.05245	0.18484	1.00000	0.43659
	0.27131	0.02831	0.11818	0.26207	0.24672	0.37118	0.33948	0.11995	0.46998	0.19034	0.27493	0.10811
12 Effective	0.34101	-0.14293	0.51858	0.37941	0.33603	-0.06862	-0.16334	0.57457	-0.08962	0.47481	0.43659	1.00000
	0.25914	-0.12619	0.42090	0.56971	0.55846	0.40454	0.58242	0.21138	0.40760	0.17944	0.43633	0.29925
13 Profound	0.17932	0.01078	0.23059	0.40604	0.05277	-0.04074	0.09002	0.27448	0.00316	0.06740	0.27131	0.25914
	1.00000	0.04781	0.11670	0.17206	0.14559	0.39311	0.25461	0.06040	0.42431	0.32371	0.35802	0.09435

14 Simple	-0.18006	0.16971	-0.00134	-0.07720	-0.05650	0.35183	0.24103	-0.14837	0.28390	0.02136	0.02831	-0.12619
	0.04781	1.00000	-0.03317	0.00006	-0.06145	-0.05588	-0.06170	0.07098	-0.04820	0.10100	-0.08840	0.03002
15 Concerned	0.20436	-0.08923	0.38474	0.13418	0.29997	0.09027	-0.25763	0.23532	-0.01664	0.72002	0.11818	0.42090
	0.11670	-0.03317	1.00000	0.69765	0.51551	0.13489	0.37649	0.25801	0.13209	-0.05497	0.41256	0.49649
16 Humane	0.21243	-0.07562	0.50338	0.20220	0.41362	0.11231	-0.27665	0.34434	-0.02408	0.66607	0.26207	0.56971
	0.17206	0.00006	0.69765	1.00000	0.60664	0.22010	0.47903	0.27114	0.20824	-0.01632	0.46331	0.51971
17 Motivating	0.14418	-0.09964	0.36227	0.11365	0.31630	0.00051	-0.25542	0.29557	-0.06813	0.49481	0.24672	0.55846
	0.14559	-0.06145	0.51551	0.60664	1.00000	0.19007	0.46491	0.33225	0.19246	0.00857	0.39368	0.49751
18 Analytical	0.42960	-0.20783	0.41474	0.55420	0.15521	-0.09738	-0.05918	0.42188	-0.00716	0.07539	0.37118	0.40454
	0.39311	-0.05588	0.13489	0.22010	0.19007	1.00000	0.41482	0.06383	0.51355	0.31448	0.28163	0.00975
19 Knowledgeab	0.45603	-0.19373	0.35462	0.38408	0.17123	-0.01790	-0.14746	0.44228	-0.07693	0.36108	0.33948	0.58242
	0.25461	-0.06170	0.37649	0.47903	0.46491	0.41482	1.00000	0.22420	0.35430	0.15069	0.52200	0.28982
20 Humorous	0.08795	-0.05522	0.22199	0.05740	0.22732	0.25119	-0.27991	0.01958	-0.06852	0.27555	0.11995	0.21138
	0.06040	0.07098	0.25801	0.27114	0.33225	0.06383	0.22420	1.00000	0.01891	-0.11420	0.15362	0.53756
21 Exacting	0.29710	-0.11401	0.26814	0.35350	0.20262	-0.18404	0.04706	0.33554	-0.00863	0.04285	0.46998	0.40760
	0.42431	-0.04820	0.13209	0.20824	0.19246	0.51355	0.35430	0.01891	1.00000	0.51823	0.36963	0.05600
22 Rigorous	0.16065	-0.00476	0.05753	0.26112	0.08867	-0.09930	0.16632	0.20489	0.12362	-0.15746	0.19034	0.17944
	0.32371	0.10100	-0.05497	-0.01632	0.00857	0.31448	0.15069	-0.11420	0.51823	1.00000	0.21179	-0.09529
23 Enlightened	0.30276	-0.12306	0.32306	0.26595	0.14681	-0.05840	-0.08734	0.36140	-0.05419	0.33226	0.27493	0.43633
	0.35802	-0.08840	0.41256	0.46331	0.39368	0.28163	0.52200	0.15362	0.36963	0.21179	1.00000	0.34047
24 Warm	0.07474	0.05676	0.22534	-0.01909	0.34277	0.12354	-0.37224	0.07638	0.00078	0.54008	0.10811	0.29925
	0.09435	0.03002	0.49649	0.51971	0.49751	0.00975	0.28982	0.53756	0.05600	-0.09529	0.34047	1.00000

Table 2

Eigenvalues for First-Order Analysis

Variables	Eigenvalues
1	6.60609
2	3.10131
3	1.87082
4	1.18534
5	1.08376
6	1.01812
7	0.96067
8	0.82819
9	0.78923
10	0.73786
11	0.69134
12	0.62247
13	0.57061
14	0.53843
15	0.47003
16	0.45479
17	0.42017
18	0.39013
19	0.34574
20	0.30675
21	0.29000
22	0.27659
23	0.25382
24	0.18774

Table 3

First-Order Principal Components Pattern/Structure Matrix and h^2

Variables	Factors						h^2
	I	II	III	IV	V	VI	
1	0.51523	-0.30896	-0.20748	0.35048	-0.00547	-0.24427	0.58649
2	-0.21982	0.11050	0.45469	-0.16208	-0.13961	0.12283	0.32812
3	0.67640	-0.04240	0.00204	0.33686	-0.17469	0.12246	0.61830
4	0.52508	-0.48196	0.05650	0.35249	-0.09473	0.03851	0.64590
5	0.49114	0.20269	0.01245	0.17140	-0.02548	0.63937	0.72128
6	-0.03212	0.32509	0.52468	0.47626	0.11710	-0.15515	0.64661
7	-0.29660	-0.45569	0.44131	0.03683	-0.24539	-0.28243	0.63173
8	0.62144	-0.26451	-0.17713	0.07700	-0.27008	0.08567	0.57374
9	-0.10176	-0.06869	0.56777	-0.12694	-0.30090	0.14040	0.46381
10	0.63817	0.53222	0.04011	-0.01848	-0.24008	0.02957	0.75099
11	0.49836	-0.29985	0.16396	-0.01976	0.10954	0.08132	0.38416
12	0.79435	-0.05498	-0.04111	-0.05451	-0.13076	0.02183	0.65626
13	0.41048	-0.40203	0.26123	-0.15850	0.22116	-0.09136	0.48074
14	-0.10942	0.06450	0.73843	0.16794	0.00001	-0.04558	0.59170
15	0.64379	0.43347	0.06355	-0.12372	-0.24043	-0.12993	0.69640
16	0.75477	0.36278	0.10726	-0.08282	-0.20893	-0.02519	0.76393
17	0.66474	0.32110	0.01652	-0.22608	0.00838	-0.05138	0.59908
18	0.55989	-0.50135	-0.00898	0.20085	0.11177	0.05103	0.62036
19	0.71823	-0.09250	-0.02882	-0.01430	0.03611	-0.36007	0.65640
20	0.35832	0.41899	0.14548	0.23877	0.60881	-0.01471	0.75299
21	0.51820	-0.52506	0.10948	-0.26780	0.22749	0.18243	0.71296
22	0.21196	-0.56770	0.27031	-0.29146	0.16595	0.23587	0.60840
23	0.64161	-0.06695	0.04837	-0.32538	0.05262	-0.35477	0.65299
24	0.50631	0.57267	0.15781	-0.16881	0.28809	0.03765	0.72211

Table 4

Promax-Rotated Factor Pattern Matrix

Variable	Factor					
	I	II	III	IV	V	VI
1	-0.02169	-0.71647	-0.19015	0.06500	0.07318	-0.13953
2	0.12826	0.31334	0.48452	-0.08565	-0.04537	0.05279
3	0.25890	-0.64470	0.04700	0.06368	0.04026	0.26581
4	-0.12367	-0.77168	0.09409	-0.16557	0.00695	0.10439
5	0.15269	-0.22425	-0.01878	-0.12821	0.08830	0.75637
6	0.00751	-0.19652	0.49667	0.30257	0.59502	-0.02732
7	-0.19785	-0.15510	0.55929	-0.05180	-0.16375	-0.39318
8	0.25373	-0.53321	-0.10036	-0.07920	-0.28102	0.14018
9	0.16537	0.12209	0.65302	-0.14336	-0.16942	0.05900
10	0.84798	-0.06441	0.06669	0.23992	-0.00218	0.16733
11	0.06118	-0.26115	0.08107	-0.44390	0.08030	0.07994
12	0.52254	-0.33689	-0.05283	-0.18127	-0.10493	0.08911
13	0.02436	-0.11268	0.13150	-0.59205	0.12267	-0.15208
14	0.01732	0.00415	0.72816	-0.04634	0.33111	-0.04661
15	0.89559	-0.02297	0.08525	0.17171	-0.05431	-0.03007
16	0.85451	-0.11531	0.10977	0.06002	-0.01731	0.08335
17	0.72720	0.08352	-0.06499	-0.11209	0.05128	0.01432
18	-0.16051	-0.60285	-0.06395	-0.37688	0.08223	0.08733
19	0.46301	-0.34012	-0.08670	-0.11877	0.08170	-0.29440
20	0.05940	0.01051	-0.11090	-0.09687	0.81106	0.13525
21	-0.04295	-0.10634	-0.03665	-0.80903	-0.02784	0.09660
22	-0.19885	0.02679	0.16153	-0.80484	-0.07934	0.09633
23	0.59444	0.00436	-0.04237	-0.30618	-0.02541	-0.36932
24	0.60843	0.31059	-0.02877	-0.13259	0.40474	0.12906

Table 5

First-Order Factor Structure Matrix

Var.	Factors					
	I	II	III	IV	V	VI
1	0.25252	-0.72363	-0.29105	-0.26257	0.08840	-0.20610
2	-0.08142	0.30906	0.49400	0.06442	-0.05330	0.10363
3	0.50227	-0.67480	-0.09962	-0.23560	0.13087	0.25448
4	0.17992	-0.77491	-0.00027	-0.42971	-0.02819	-0.00726
5	0.41058	-0.26079	-0.08809	-0.16519	0.16106	0.74441
6	0.03486	-0.01175	0.44697	0.28384	0.56635	0.03031
7	-0.37797	-0.06459	0.58371	-0.06076	-0.28186	-0.43533
8	0.42315	-0.65033	-0.22099	-0.37163	-0.19002	0.10416
9	-0.04009	0.10742	0.63041	-0.06894	-0.17987	0.08982
10	0.81244	-0.22850	-0.12520	0.00854	0.25125	0.32383
11	0.28745	-0.44857	-0.02138	-0.54844	0.08390	0.01222
12	0.68401	-0.58861	-0.23903	-0.46214	0.05221	0.10795
13	0.20465	-0.36483	0.03435	-0.64816	0.09643	-0.22724
14	-0.06505	0.07491	0.69075	0.02888	0.27214	-0.01646
15	0.81469	-0.24743	-0.12080	-0.08065	0.20134	0.12748
16	0.85775	-0.35826	-0.11052	-0.20125	0.22643	0.21340
17	0.76269	-0.22020	-0.25012	-0.28023	0.26260	0.11940
18	0.20205	-0.70676	-0.15315	-0.57661	0.04051	-0.04265
19	0.61196	-0.58956	-0.27859	-0.43189	0.21210	-0.27093
20	0.36486	-0.06305	-0.19795	-0.07800	0.83943	0.16004
21	0.23148	-0.42421	-0.11854	-0.83364	-0.05685	-0.02666
22	-0.03550	-0.20701	0.14244	-0.71260	-0.17139	-0.03375
23	0.62371	-0.36756	-0.22995	-0.52034	0.12646	-0.31947
24	0.68247	0.05080	-0.17992	-0.14431	0.57964	0.24939

Table 6

First-Order Factor Correlation Matrix

I	1.00000					
II	-0.34165	1.00000				
III	-0.25003	0.14358	1.00000			
IV	-0.27462	0.41645	0.10189	1.00000		
V	0.28851	-0.01235	-0.08345	0.03041	1.00000	
VI	0.15323	0.09625	0.02842	0.12899	0.03738	1.00000

Table 7

Eigenvalues for Second-Order Analysis

Factors	Eigenvalues
I	1.83302
II	1.25257
III	0.95428
IV	0.88463
V	0.57924
VI	0.49626

Table 8

Second-Order Factor Pattern/Structure Matrix

First-Order Factors	Second-Order Factors A	Second-Order Factors B
I	0.74760	0.38289
II	-0.72913	0.30805
III	-0.46220	-0.16502
IV	-0.66508	0.42795
V	0.28940	0.62338
VI	-0.05251	0.64195

Table 9

Varimax Rotated Second-Order Factor Pattern/Structure Matrix and h^2

Factor	A	B	h^2
I	0.51462	0.66384	0.70552
II	-0.79096	-0.03029	0.62653
III	-0.34858	-0.34548	0.24086
IV	-0.78380	0.10545	0.62547
V	-0.00232	0.68728	0.47235
VI	-0.31983	0.55907	0.41485
Trace	1.72860	1.35700	3.08560

From Thompson, B. (1990). SECONDOR: A program that computes a Second-order principal components analysis and various interpretation aids. Educational and Psychological Measurement, 50, 577. Reprinted with permission of the author.

Table 10

Product Matrix (P_{vxh}) and h^2

Var.	A	B	h^2
1	0.61533	0.05214	0.38135
2	-0.30037	-0.10244	0.10072
3	0.49176	0.35815	0.37010
4	0.61030	-0.04555	0.37454
5	0.12087	0.58467	0.35645
6	-0.24362	0.26493	0.12954
7	-0.00737	-0.65769	0.43261
8	0.60520	0.09614	0.37551
9	-0.14520	-0.21810	0.06865
10	0.22253	0.65919	0.48405
11	0.53196	0.07358	0.28839
12	0.66761	0.33393	0.55721
13	0.56823	-0.08899	0.33080
14	-0.19773	-0.04358	0.04099
15	0.32450	0.52974	0.38593
16	0.41903	0.57386	0.50490
17	0.41399	0.53410	0.45665
18	0.68380	-0.00060	0.46758
19	0.72457	0.22665	0.57638
20	0.09170	0.70024	0.49875
21	0.67807	-0.06308	0.46376
22	0.42038	-0.27416	0.25189
23	0.67540	0.15289	0.47954
24	0.13918	0.74077	0.56811
Trace	5.25000	3.69440	8.94440

Table 11

Varimax Rotated Product Matrix and h^2

Var.	A	B	h^2
1	0.603	0.135	0.381
2	-0.248	-0.142	0.101
3	0.439	0.421	0.370
4	0.611	0.037	0.375
5	0.041	0.596	0.356
6	-0.277	0.230	0.130
7	0.082	-0.653	0.433
8	0.587	0.177	0.376
9	-0.114	-0.236	0.069
10	0.131	0.683	0.484
11	0.517	0.145	0.288
12	0.616	0.421	0.557
13	0.575	-0.011	0.331
14	-0.190	-0.070	0.041
15	0.250	0.569	0.386
16	0.338	0.625	0.505
17	0.338	0.585	0.457
18	0.678	0.092	0.468
19	0.687	0.323	0.576
20	-0.004	0.706	0.499
21	0.680	0.029	0.464
22	0.454	-0.215	0.252

23	0.649	0.243	0.480
24	0.038	0.753	0.568
Trace	4.796	4.149	8.944

From Thompson, B. (1990). SECONDOR: A program that computes a Second-order principal components analysis and various interpretation aids. Educational and Psychological Measurement, 50, p.578. Reprinted with permission of the author.

Table 12

Schmid-Leiman Solution

Variable	A	B	I	II	III	IV	V	VI	h^2
1 Intelligent	0.615	0.052	-0.012	-0.438	-0.166	0.040	0.053	-0.107	0.616
2 Undirected	-0.300	-0.102	0.070	0.191	0.422	-0.052	-0.033	0.040	0.326
3 Honest	0.492	0.358	0.140	-0.394	0.041	0.039	0.029	0.203	0.590
4 Scholarly	0.610	-0.046	-0.067	-0.472	0.082	-0.101	0.005	0.080	0.625
5 Personable	0.121	0.585	0.083	-0.137	-0.016	-0.078	0.064	0.579	0.727
6 Easy	-0.244	0.265	0.004	-0.120	0.433	0.185	0.432	-0.021	0.553
7 Distant	-0.007	-0.658	-0.107	-0.095	0.487	-0.032	-0.119	-0.301	0.796
8 Informed	0.605	0.096	0.138	-0.326	-0.087	-0.048	-0.204	0.107	0.564
9 Docile	-0.145	-0.218	0.090	0.075	0.569	-0.088	-0.123	0.045	0.431
10 Caring	0.223	0.659	0.460	-0.039	0.058	0.147	-0.002	0.128	0.739
11 Systematic	0.532	0.074	0.033	-0.160	0.071	-0.272	0.058	0.061	0.401
12 Effective	0.668	0.334	0.284	-0.206	-0.046	-0.111	-0.076	0.068	0.705
13 Profound	0.568	-0.089	0.013	-0.069	0.115	-0.362	0.089	-0.116	0.502
14 Simple	-0.198	-0.044	0.009	0.003	0.634	-0.028	0.241	-0.036	0.504
15 Concerned	0.325	0.530	0.486	-0.014	0.074	0.105	-0.039	-0.023	0.641
16 Humane	0.419	0.574	0.464	-0.070	0.096	0.037	-0.013	0.064	0.740
17 Motivating	0.414	0.534	0.395	0.051	-0.057	-0.069	0.037	0.011	0.624
18 Analytical	0.684	-0.001	-0.087	-0.368	-0.056	-0.231	0.060	0.067	0.675
19 Knowledgeable	0.725	0.227	0.251	-0.208	-0.076	-0.073	0.059	-0.225	0.748
20 Humorous	0.092	0.700	0.032	0.006	-0.097	-0.059	0.589	0.103	0.870
21 Exacting	0.678	-0.063	-0.023	-0.065	-0.032	-0.495	-0.020	0.074	0.721
22 Rigorous	0.420	-0.274	-0.108	0.016	0.141	-0.493	-0.058	0.074	0.535
23 Enlightened	0.675	0.153	0.323	0.003	-0.037	-0.187	-0.018	-0.283	0.700
24 Warm	0.139	0.741	0.330	0.190	-0.025	-0.081	0.294	0.099	0.817
Trace	5.25	3.69	1.27	1.06	1.46	0.92	0.79	0.71	15.15

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